Roll.No. 24PAMET4A04

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - NOVEMBER 2025
SEMESTER - IV

24PAMET4A04 - Calculus of Variations and Integral Equations

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Describe the curve $y(x)$ that extremizes the functional $v[y(x)] = \int_0^{\frac{\pi}{2}} [(y')^2 - y^2] dx$ with the boundary conditions $y(0) = 0$ and $y(\frac{\pi}{2}) = 1$.
2. Compute the solution to the homogeneous Fredholm integral equation $g(s) = \lambda \int_0^1 e^s e^t g(t) dt.$
3. Solve the integral equation $g(s) = 1 + \lambda \int_0^{\pi} [\sin(s+t)] g(t) dt.$
4. Relate the properties of a symmetric kernel to the orthogonality of its eigenfunctions corresponding to different eigenvalues.
5. Explain the extremals for the functional V $= \int_{x_0}^{x_1} (y^{'2} + z^{'2} + 2yz) dx$, with the given fixed initial conditions $y(0) = 0, z(0) = 0$, and the moving endpoint condition.
6. Solve the Fredholm integral equation of the second kind g(s) $= s + \lambda \int_0^1 (st^2 + s^2 t)\ g(t)\ dt.$
7. Discuss Fredholm's first theorem.
8. Determine the solution to the integral equation $S = \int_0^s \frac{g(t)}{(s-t)^{\frac{1}{2}}} dt$

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Prove that the shortest path on which a particle in the absence of friction will side from one point to another in the shortest time under the action of gravity is a cycloid.

Contd...

10. i) Classify whether the Jacobian condition is fulfilled for the extremal curve of the functional.

    $V[y(x)] = \int_0^a (y' + y^2 + x^2)dx$,

    given that the curve is fixed at the boundary points A(0, 0) and B(a, 0).

    ii) List the conditions that are sufficient for a functional V to achieve an extremum on the curve C.

11. Ascertain the solution for the integral $g(s)$ equation.

    $g(s) = f(s) + \lambda \int_0^1 (1 - 3st)g(t)dt.$

12. Generate the Neumann series for the solution of the integral equation.

    $g(s) = (1 + s) + \lambda \int_0^s (s - t)g(t)dt.$

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. Justify the Hilbert-Schmidt theorem by stating the theorem and providing its rigorous proof.

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