

Roll.No.

20PAMCT4010

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.

M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - NOVEMBER 2025
SEMESTER - IV

20PAMCT4010 - Functional Analysis

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Let N and N' be normed linear spaces and T a linear transformation of N into N' . Then identify the following conditions on T are all equivalent to one another:
(i) T is continuous;
(ii) T is continuous at the origin, in the sense that $x_n \rightarrow 0 \Rightarrow T(x_n) \rightarrow 0$;
(iii) there exists a real number $k \geq 0$ with the property that $\|T(x)\| \leq k\|x\|$ for every $x \in N$.
(iv) if $S = \{x : \|x\| \leq 1\}$ is the closed unit sphere in N , then its image $T(S)$ is a bounded set in N' .
2. If T is an operator on H for which $(Tx, x) = 0$, for all x , then interpret $T = 0$
3. Classify the mapping $x \rightarrow x^{-1}$ of G into G is continuous and is therefore a homeomorphism of G onto itself.
4. If f_1 and f_2 are multiplicative functional on A with the same null space M , then illustrate $f_1 = f_2$.
5. Let $\{e_1, e_2, \dots, e_n\}$ be a finite orthonormal set in a Hilbert space H . If x is any vector in H , then show that $\sum_{i=1}^n |(x, e_i)|^2 \leq \|x\|^2$; further, $x - \sum_{i=1}^n (x, e_i)e_i \perp e_j$ for each j .
6. If T is an operator on H , then predict T is normal \Leftrightarrow its real and imaginary parts commute.
7. If $1 - xr$ is regular, then show that $1 - rx$ is also regular.
8. If x is a normal element in a B^* - algebra, then justify $\|x^2\| = \|x\|^2$.

Contd...

Section C

I - Answer any **TWO** questions ($2 \times 10 = 20$ Marks)

9. State and prove Hahn-Banach theorem.
10. State and prove closed graph theorem.
11. Justify the adjoint operations $T \rightarrow T^*$ on $B(H)$ has the following properties:
 - (i) $(T_1 + T_2)^* = T_1^* + T_2^*$
 - (ii) $(\alpha T)^* = \bar{\alpha}T^*$
 - (iii) $(T_1T_2)^* = T_2^*T_1^*$
 - (iv) $T^{**} = T$
 - (v) $\|T^*\| = \|T\|$
 - (vi) $\|T^*T\| = \|T\|^2$
12. Prove $r(x) = \lim \|x^n\|^{\frac{1}{n}}$.

II - Compulsory question ($1 \times 10 = 10$ Marks)

13. Examine the Gelfand-Neumark theorem.
