

Roll.No.

20PAMCT3008

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.

M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - NOVEMBER 2025
SEMESTER - III

20PAMCT3008 - Differential Equations

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Solve $x''(t) + k \sin x(t) = 0$ with initial conditions $x(0) = \frac{\pi}{6}$ and $x'(0) = 0$. where k is a constant.
2. Prove that $\int_{-1}^1 p_n^2(t) dt = \frac{2}{2n+1}$. if p_n is a Legendre polynomial
3. Find e^{At} when $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$
4. Prove that the set of all solutions of the system $x' = A(t)x$ on I forms an n -dimensional vector space over the field of complex numbers.
5. Compute the first three successive approximations for $x' = x^2$, $x(0) = 1$
6. Solve $Z_x + Z_y = e^{x+y}$ with $z(x,0) = \sin x$
7. Solve $u_{xx} + u_{yy} = 0$.
8. Solve $p^2 + q^2 + r^2 = 1$ using Jacobi's method

Section C

I - Answer any **TWO** questions ($2 \times 10 = 20$ Marks)

9. Prove that $\int_1^0 t J_p(a_m t) J_p(a_n t) dt = \begin{cases} 0, & m \neq n \\ \frac{1}{2} J_{p+1}^2(a_n), & m = n, \end{cases}$
where a_1, a_2, \dots be the positive zeros of the Bessel function $J_p(t)$.
10. Find e^{At} when $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$
11. Solve $z = px + qy + p^2 + q^2$ using Charpit's method

Contd....

12 Solve

$$(i) u_{m+1,n} + u_{m,n+1} - 3u_{m,n} = 0$$

$$(ii) u_{m+1,n+1} - 2u_{m+1,n} - 3u_{m,n+1} + 6u_{m,n} = 0$$

II - Compulsory question (1 × 10 = 10 Marks)

13. Show that $f(t,x) = x^{\frac{1}{2}} + 1$ does not satisfy the Lipschitz condition. Consider the IVP $x' = x^{\frac{1}{2}} + 1, x(0) = 0$ on the rectangle $0 \leq t \leq 1$ and $0 \leq x \leq 1$.
