Roll.No. [ ] **20PAMCT1002**

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(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - NOVEMBER 2025
SEMESTER - I
20PAMCT1002 - Real Analysis

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that the class M (the class of Lebesque measurable set) is a σ algebra.
2. State and prove Lebesque's dominated convergence theorem.
3. Let α be monotonically increasing on [a,b]. Suppose $f_n \in R(\alpha)$ on[a,b] for n=1,2,3 ....and $f_n \rightarrow f$ uniformly on [a,b]. Then prove that f $\in R(\alpha)$ on [a,b] and $\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$
4. If X is a complete metric space and if ϕ is a contraction of X into X then prove that there exist one and only one $x \in X$ such that ϕ(x) $= x$
5. Let ϕ_n be orthonormal on [a,b] and if f(x) $= \sum_{n=1}^{\infty} c_n\phi_n(x)$ then prove that $\sum_{n=1}^{\infty} |c_n| \leq ||f||^2 and \lim_{n\to\infty} c_n = 0$
6. If E_i be a sequence of measurable sets, then prove that
   (a)$E \subseteq E_2 \subseteq E_3 \subseteq ....$, then $m(\lim Ei) = \lim m(Ei)$
   (b) if $E_1 \supseteq E_2 \supseteq E_3 \supseteq ....., m(Ei) < \infty$ for each interval I then
   $m(\lim Ei) = \lim(m(Ei))$
7. State and prove Implicit function theorem.
8. Prove that there exist a real continuous function on the real line which is nowhere differentiable.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. State and prove Fatou's lemma
10. (a) State and prove Cauchy criterion for uniform convergence.
    (b) State and prove Weierstrass M-test for uniform convergence.
11. State and prove inverse function theorem.

Contd...

12. Suppose f and g are Riemann -Integrable functions with period 2π and

$$f(x)=\sum_{-\infty}^{\infty} c_n e^{inx}, g(x) = \sum_{-\infty}^{\infty} \gamma_n e^{inx}$$

Then prove that $\lim_{N\to\infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} |f(x) - S_N(f;x)|^2 dx = 0$

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. Prove that Lebesque outer measure of an interval equals to its length

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