

Roll.No.

Course Code

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc PBPB - END SEMESTER EXAMINATIONS - NOVEMBER 2025

SEMESTER - VI

20UMACT6014 -Complex Analysis

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions (6 × 5 = 30 Marks)

- (i) Define Limit of a function.
(ii) Show that the function $f(z) = \bar{z}$ is nowhere differentiable.
- Prove that $v(x,y) = y^4 + x^4 - 6x^2y^2$ is harmonic and find its harmonic conjugate.
- Using Cauchy's integral formula, evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ Where C is the circle $|z| = 3$
- show that $\frac{1}{z^2} = \frac{1}{4} + \frac{1}{4} \sum_{n=1}^{\infty} (-1)^n (n+1) \left(\frac{z-2}{2}\right)^n$
- (i) State Morera's theorem.
(ii) Evaluate $\int_C \frac{e^{2z}}{(z+1)} dZ$.
- State and prove Cauchy's inequality.
- If $f(z)$ and $\overline{f(z)}$ are analytic in a region D, then show that $f(z)$ is a constant in that region.
- verify Cauchy-Riemann equation for the function $f(Z) = Z^3$ and find $f'(Z)$.

Section C

Answer any **THREE** questions (3 × 10 = 30 Marks)

- State and prove a sufficient condition for $f(z)$ to be analytic.
- Derive the Cauchy-Riemann equations in polar coordinates.
- State and prove Cauchy's integral formula.
- State and prove Liouville's theorem and hence deduce fundamental theorem of algebra.
- Find the Laurent series expansion of $\frac{z^2 - 1}{z^2 + 5z + 6}$ in the region $2 < |z| < 3$
