

Roll.No.

20USTAT2002

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc Statistics - END SEMESTER EXAMINATIONS - NOVEMBER 2025

SEMESTER - II

20USTAT2002 - Allied Mathematics-II

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Consider the function f defined by $f(x) = x^2, -2025 < x < \infty$
 - (i) What is the domain of f ?
 - (ii) What is the range of f ?
 - (iii) What is the image of 2025 under f ?
 - (iv) Find $f^{-1}(2025)$.
2. If $\lim_{n \rightarrow \infty} S_n = L$ and $\lim_{n \rightarrow \infty} S_n = M$, then prove that $L = M$.
3. If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{S_n\}_{n=1}^{\infty} = 1$ is bounded. What about the converse part? Justify.
4. If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B , then prove that $\sum_{n=1}^{\infty} (a_n + b_n)$ converges to $A + B$.
5. Evaluate $L(e^t + \frac{1}{e^t})^2$
6. Find the Laplace transform of $\sin 3t \cos t$.
7. Find the value of $L(t \cos at)$.
8. State and prove First shifting property of Laplace transform.

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. Prove that the set of all real numbers R is uncountable.
10. Prove that
 - (i) A non decreasing sequence which is bounded above is convergent.
 - (ii) A non decreasing sequence which is not bounded above diverges to infinity.

Contd...

11. If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that $a_1 \geq a_2 \geq \dots \geq a_n \geq a_{n+1} \geq \dots$, and $\lim_{n \rightarrow \infty} a_n = 0$, then prove that the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is convergent.

12. Find the Laplace transform of $f(t)$ if

$$f(t) = \begin{cases} e^{-t}, & 0 \leq t \leq 4 \\ 0, & 4 < t < \infty \end{cases}$$

13. Find the Laplace transform of $e^{-t} \int_0^t \frac{\sin^t}{t} dt$.
