

Roll.No.

20USTAT2002

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)  
Chromepet, Chennai - 600 044.

B.Sc Statistics - END SEMESTER EXAMINATIONS - NOVEMBER 2025  
SEMESTER - II

20USTAT2002 - Allied Mathematics-II

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

### Section B

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. If  $f: A \rightarrow B$  and if  $X \subset B, Y \subset B$  then prove that  
 $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$
2. If the sequence of real numbers  $\{s_n\}_{n=1}^{\infty}$  is convergent, then prove that  $\{s_n\}_{n=1}^{\infty}$  is bounded.
3. Let  $\varphi$  be a continuous function on the closed bounded interval  $[a, b]$ , and let  $g$  be a continuous function on  $[a, b]$  such that  $g(t) \geq 0, (a \leq t \leq b)$ . Then prove that there exists a number  $c$  with  $a \leq c \leq b$  such that  $\int_a^b \varphi(t)g(t)dt = \varphi(c) \int_a^b g(t)dt$ .
4. Find (i)  $L(\cos 3t \cos t)$ , (ii)  $L(e^{-5t} \cos^2 t)$ .
5. Find (i)  $L^{-1}\left(\frac{s^3}{(s-4)^4}\right)$  (ii)  $L^{-1}\left(\frac{s}{s^2+2s+10}\right)$
6. Show that the series  $\sum_{n=1}^{\infty} \frac{1}{n}$  is divergent.
7. Prove that the set of rational numbers is countable.
8. Evaluate  $L^{-1}\left[\log\left(\frac{s}{(s^2+4)^2}\right)\right]$

### Section C

Answer any **THREE** questions ( $3 \times 10 = 30$  Marks)

9. Prove that the countable union of countable sets is countable.
10. Prove that the sequence  $\left\{\left(1 + \frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$  is convergent.
11. State and prove Rolle's theorem.
12. (i) Find the Laplace transform of  $e^{-t} \int_0^t \frac{\sin t}{t} dt$ .  
(ii) Evaluate  $L^{-1}\left(\frac{\cos hat - \cos hbt}{t}\right)$

Contd...

13. (i) Evaluate  $L^{-1}\left(\frac{5s+3}{(s-1)(s^2+2s+5)}\right)$

(ii) Show that  $L^{-1}\left(\log\left(1+\frac{w^2}{s^2}\right)\right)=\frac{2(1-\cos wt)}{t}$

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