Roll.No. ☐ **20UMACT6014**

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc Mathematics - END SEMESTER EXAMINATIONS - NOVEMBER 2025
SEMESTER - VI
20UMACT6014 -Complex Analysis

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. (i) Define Limit of a function.
   (ii) Show that the function $f(z) = \overline{z}$ is nowhere differentiable.
2. Prove that v(x,y)=$y^4 + x^4 - 6x^2y^2$is harmonic and find its harmonic conjugate.
3. Using Cauchy's integral formula, evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\ dz$ where c is the circle $|z| = 3$.
4. show that $\frac{1}{z^2}$=$\frac{1}{4} + \frac{1}{4}\sum_{n=1}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n$ when $|z-2| < 2$.
5. (i) State Morera's theorem.
   (ii) Evaluate $\int \frac{e^{2z}}{(z+1)} dZ$.
6. State and prove Cauchy's inequality.
7. If f(z) and $\overline{f(z)}$ are analytic in a region D,then show that f(z) is a constant in that region.
8. verify Cauchy-Riemann equation for the function $f(z) = z^3$ and find $f'(z)$.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. State and prove a sufficient condition for f(z) to be analytic.
10. Derive the Cauchy-Riemann equations in polar coordinates.
11. State and prove Cauchy's integral formula.
12. State and prove Liouville's theorem and hence deduce fundamental theorem of algebra.
13. Find the Laurent series expansion of $\frac{z^2-1}{z^2+5z+6}$ in the region $2 < |z| < 3$

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