Roll.No. [ ] **20UMACT6013**

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc Mathematics- END SEMESTER EXAMINATIONS - NOVEMBER 2025
SEMESTER - VI
20UMACT6013 - Linear Algebra

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If $\{v_1, v_2, v_3 v_n\}$ is a basis of a vector space V over F and if $\{w_1, w_2, w_3 w_m\} \in V$are linearly independent over F, then prove that m $\leq$ n.
2. If V is finite -dimensional and $v \neq 0 \in V$, then prove that there is an element $f \in v$ such that $f(v) \neq 0$
3. Derive Cauchy – Schwarz inequality.
4. Prove that the element $\lambda \in F$ is a characteristics root of $T \in A(v)$ if and only if for some $v \neq 0$ in V,vT=λv
5. If V is finite dimensional over F, prove that $T \in A(V)$ is singular if and only if there exists $v \neq 0$ such that vT = 0.
6. Prove that the vectors (1, 2, 1), (2, 1, 0) and(1, –1, 2) are linearly independent.
7. Prove that $F^{(n)}$is isomorphic $F^{(m)}$if and only if n=m.
8. If T,$S \in A(V)$and if S is regular, prove that T and STS^{-1}have the same minimal polynomial.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If V is finite dimensional and if W is a subspace of V, then prove that (a) dimW $\leq$ dimV and (b) dim(V/W) = dimV - dimW .
10. If V and W are of dimensions m and n respectively over F then prove that Hom(V,W)is of dimension mn over F .
11. Let V be a finite dimensional inner product space then prove that V has an orthonormal set as a basis.

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12. If V is finite-dimensional over F , prove that T $\in$ A(v) is invertible if and only if the constant term of the minimal polynomial for T is not zero.
13. If λ F is a characteristic root of T $\in A(V)$ prove that λ is a root of the minimal polynomial of T.

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