Roll.No.

20UMACT5010

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc Mathematics- END SEMESTER EXAMINATIONS - NOVEMBER 2025
SEMESTER - V
20UMACT5010 - Real Analysis

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that the set $[0,1] = \{x|0 \leq x \leq 1\}$ is uncountable.
2. If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that
(i) If $\{a_n\}n^{\infty} = 1$ is non increasing and
(ii)$\lim_{n\to\infty} a_n = 0$, then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ is convergent.
3. If f and g are real - valued functions, if f is continuous at a, and g is continuous at $f(a)$, prove that $g \circ f$ is continuous at a.
4. Let (M, ρ) be a complete metric space. if T is contraction on M, then prove that there is one and only one point x in M such that Tx = x.
5. State and prove the comparison test for absolute convergence.
6. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.
7. Let M be a metric space. Prove that M is connected if and only if every continuous characteristic function on M is constant.
8. State and Prove Rolle's theorem. Also show that $f(x) = x^2 \; 0 < x < 1$ satisfies Rolle's theorem.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. i) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then $\{S_n\}_{n=1}^{\infty}$ cannot also converge to a limit distinct from L. That is, if $\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} S_n = M$ then prove that $L = M$.
ii) If A_1, A_2, ... are countable sets prove that $\bigcup_{n=1}^{\infty} A_n$ is also countable.

Contd...

10. i) If $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers, then prove that $\{S_n\}_{n=1}^{\infty}$ is convergent.

    ii) Prove that the series $\sum_{n=1}^{\infty}(\frac{1}{n})$ is divergent.

11. Define : (i)Metric space, (ii)Absolute value metric (iii) Discrete metric.

    ii) Let E be a subset of the metric space M.Then prove that the point x $\in$ M is a limit point of E if and only if every open Ball B[x:r] about x contains at least one point of E.

12. Let (M, ρ) be a metric space. Prove that the subset A of M is totally bounded if and only if every sequence of points of A contains a Cauchy subsequence.

13. State and Prove first Fundamental theorem of Calculus.

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