

Roll.No.

20UMACT5009

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.

B.Sc Mathematics- END SEMESTER EXAMINATIONS - NOVEMBER 2025
SEMESTER - V

20UMACT5009 - Modern Algebra

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Show that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G .
2. If H is a nonempty finite subset of a group G and H is closed under multiplication, then show that H is a subgroup of G .
3. If ϕ is a homomorphism of G into \overline{G} with kernel k , then show that kernel k is a normal subgroup of G .
4. State and Prove Cayley's theorem.
5. Define Ring. Show that the set of all real quaternions is a ring.
6. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Show that R is a field.
7. Show that the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R .
8. Let R be a Euclidean ring. Show that any two elements a and b in R have a greatest common divisor d . Also show that $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$.

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. If H and K are finite subgroups of G of order $o(H)$ and $o(K)$ respectively, then show that $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$.
10. If ϕ is a homomorphism of G into \overline{G} with kernel K , show that $\frac{G}{K} \approx \overline{G}$.
11. Prove that a finite integral domain is a field.
12. If U is an ideal of the ring R , then prove that R/U is a ring and is a Homomorphic image of R .
13. Prove that every integral domain can be embedded in a field.
