

Roll.No.

20UMACT4007

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)  
Chromepet, Chennai - 600 044.

B.Sc Mathematics- END SEMESTER EXAMINATIONS - NOVEMBER 2025  
SEMESTER - IV

**20UMACT4007 - Vector Calculus and Fourier Transforms**

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

### Section B

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. Find a unit normal to the surface  $x^2y + 2xz^2 = 8$  at the point  $(1, 0, 2)$ .
2. Find the directional derivative of  $\phi = x^2yz + 4xz^2 + xyz$  at  $(1, 2, 3)$  in the direction of  $2i + j - k$ .
3. Prove that the area bounded by a simple closed curve C is given by  $\frac{1}{2} \int_c (x dy - y dx)$ .
4. If  $F = (3x^2 + 6y)i - 14yzj + 20xz^2k$ , evaluate  $\int_c F \cdot dr$  from  $(0, 0, 0)$  to  $(1, 1, 1)$  along the curve  $x = t, y = t^2, z = t^3$ .
5. Verify Stoke's theorem for  $F = (2x - y)i - yz^2j - y^2zk$  where s is the upper half of the sphere  $x^2 + y^2 + z^2 = 1$  and c is the circular boundary on  $z = 0$  plane.
6. Find the Fourier cosine transform of  $\frac{1}{a^2 + x^2}$ .

7. If  $f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$  and

$$F(S) = \frac{2 \sin as}{s}, s \neq 0,$$

Prove that  $\int_0^\infty \frac{\sin^2 ax}{x^2} dx = \frac{\pi a}{2}$ .

8. Find the Fourier transform of

$$f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a > 0 \end{cases}$$

Deduce that  $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$

### Section C

Answer any **THREE** questions ( $3 \times 10 = 30$  Marks)

9. If  $r$  is the position vector of the point  $P(x, y, z)$ ,  
Prove (i)  $\text{div } r = 3$  (ii)  $\text{curl } r = 0$  (iii)  $\nabla r^n = nr^{n-2}r$  where  $r = |r|$ .

Contd...

10. Find the common area between  $y^2 = 4x$  and  $x^2 = 4y$  using Green's Theorem.
11. Verify Gauss's Divergence theorem for  $F = x^2i + y^2j + z^2k$  taken over the cube bounded by the planes  $x = 0, x = 1, y = 0, y = 1, z = 0$  and  $z = 1$ .

12. Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

Hence evaluate  $\int_0^\infty \left( \frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$ .

13. Using Parseval's identity evaluate  $\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$  and  $\int_0^\infty \frac{x^2 dx}{(a^2 + x^2)^2}$  if  $a > 0$

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