

Roll.No.

20UMACT2004

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.

B.Sc Mathematics- END SEMESTER EXAMINATIONS - NOVEMBER 2025
SEMESTER - II

20UMACT2004 - Integral Calculus and Fourier Series

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Use Bernoulli's formula to find the value of $\int x^2 e^{-2x} dx$
2. Show that
 - (i) $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^5 x dx = \frac{8}{693}$
 - (ii) $\int_0^{\frac{\pi}{2}} \sin^7 x dx = \frac{48}{105}$
3. Show that $\Gamma(n + 1) = n\Gamma(n)$
4. Apply the properties of Gamma function to reduce $1.3.5...(2n - 1)$ to $\frac{2^{1-n}\Gamma(2n)}{\Gamma(n)}$
5. Prove that $\beta(m, n) = \beta(m, n + 1) + \beta(m + 1, n)$
6. Simplify $\int_0^{\infty} x^4 e^{-x} dx$ using Gamma function and show that it has a value 24.
7. Illustrate how the Fourier coefficients a_0, a_n are zero for $f(x) = \frac{1}{2}(\pi - x); 0 < x < 2\pi$
8. Show that for all values of x on $(-\pi, \pi)$ we have $\frac{x}{2} = \sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \dots \infty$

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. Derive the reduction formula for $I_n = \int x^n e^{ax} dx$ Hence calculate I_3

Contd...

10. Solve $\int_0^1 x^5(1-x^3)^{10}dx$ using Beta function.

11. Derive the value of $\Gamma\left(\frac{1}{2}\right)$

12. Express $f(x) = \frac{\pi-x}{2}$ as a fourier series with period 2π to be valid in $(0, 2\pi)$.

13. Test whether $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$ is an odd function.
Hence find its Fourier expansion.
