Roll.No. | | **20UMACT2004**

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc Mathematics- END SEMESTER EXAMINATIONS - NOVEMBER 2025
SEMESTER - II
20UMACT2004- Integral Calculus and Fourier Series

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find $\int x^3 \cos 2x \ dx$.
2. Find $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^5 x \ dx$.
3. Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy \, dx \, dy$ and evaluate it.
4. Express $\int_0^1 x^m(1-x^n)^p dx$ in term of Gamma functions and evaluate the integral $\int_0^1 x^5(1-x^3)^{10} dx$.
5. Prove that $\Gamma(n+1) = n!$ where n is a positive integer.
6. Express $f(x) = x(-\pi < x < \pi)$ as a Fourier series with period 2π.
7. Express $f(x) = c - x(0 < x < c)$ as a half range cosine series with period 2c.
8. Find the sine series for $f(x) = c$ in the range 0 to π

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Solve $I_{m,n} = \int x^m (\log x)^n dx$ (where m and n are positive integers).
   Hence or otherwise evaluate $\int x^4 (\log x)^3 dx$.
10. Evaluate $\iiint xyz dx dy dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.
11. Derive the Relation between Beta and Gamma functions.

12. Show that $x^2 = \frac{\pi^2}{3} + 4\Sigma_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \leq x \leq \pi$.
    Deduce that
    (i) $\frac{1}{1^2}$ - $\frac{1}{2^2} + \frac{1}{3^2}$ - ... $= \frac{\pi^2}{12}$

    (ii) $\frac{1}{1^2}$ - $\frac{1}{2^2} + \frac{1}{3^2}$ +... $= \frac{\pi^2}{6}$

    (iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2}$ +... $= \frac{\pi^2}{8}$

13. Find the cosine series in the range 0 to π for
$$f(x) = \begin{cases} x & 0 < x \leq \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} < x \leq \pi \end{cases}$$

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