SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - IV
20PAMCT4010 - Functional Analysis

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M, then prove that there exists a functional f_0 in N^* such that $f_0(M) = 0$ and $f_0(x_0) \neq 0$.
2. State and prove the Hahn-Banach theorem.
3. State and prove the Open Mapping theorem.
4. If M is a closed linear subspace of a Hilbert space H, then prove that $H = M \oplus M^{\perp}$.
5. If A_1 and A_2 are self-adjoint operators on H, then show that their product $A_1\ A_2$ is self-adjoint $\Leftrightarrow A_1\ A_2 = A_2\ A_1$.
6. If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other, then show that $N_1 + N_2$ and $N_1\ N_2$ are normal.
7. Where Z: The set of all topological divisors of zero.
   S: The set of singular elements of a Banach Algebra.
8. Show that the maximal ideal space $\mathcal{M}$ is a compact Hausdorff space.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Let N and N' be normed linear spaces and T a linear transformation of N into N'. Then prove that the following conditions on T are all equivalent to one another:
   (i) T is Continuous.
   (ii) T is continuous at the origin, in the sense that $x_n \to 0 \Rightarrow T(x_n) \to 0$.
   (iii) There exists a real number $K \geq 0$ with the property that $\|T(x)\| \leq K\|x\|$ for every $x \in N$.
   (iv) If $S = \{x : \|x\| \leq 1\}$ is the closed unit sphere in N, then its image $T(S)$ is a bounded set in N'.

Contd...

10. If the adjoint operations $T \to T^*$ is a mapping of (H), then prove the following properties:
    (i) $(T_1 + T_2)^* = T_1^* + T_2^*$.
    (ii) $(\alpha T)^* = \bar{\alpha} T^*$.
    (iii) $(T_1 T_2)^* = T_2^* T_1^*$.
    (iv) $T^{**} = T$.
    (v) $\|T^*\| = \|T\|$.
    (vi) $\|T^* T\| = \|T\|^2$.
11. (i) If A is a division algebra, then it equals the set of all scalar multiples of the identity.
    (ii) If r is an element of R, then prove that $1 - r$ is regular.
12. State and prove the Gelfand - Neumark theorem.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove the Uniform Boundedness theorem.

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