SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - III
20PAMCT3008 - Differential Equations

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Show that if $P_n(t)$ is a Legendre polynomial then $\int_{-1}^{1} P_n^2(t)\ dt = \frac{2}{2n+1}$.

2. Solve the Hermite equation $x'' - 2tx' + 2nx = 0$ using power series method.

3. Find the solution of the initial valued problem $x'' - 2x' + x = 0$, $x(0) = 0$ and $x^1(0) = 1$, where $t \in [0, a] = I$ and a is finite positive real number.

4. Prove that, a solution matrix ϕ of the matrix differential equation $\overline{X}' = A(t)\ \overline{X}(t)$, $t \in I$, is a fundamental matrix of $\overline{x}' = A(t)\overline{x}$ on I if and only if $det\ \phi(t) \neq 0$, $t \in I$.

5. State and prove Gronwall's inequality.

6. Solve the $IVP x' = -x$, $x(0) = 1$, $t \geq 0$ by the method of successive approximations.

7. If $u = f(x + iy) + g(x - iy)$, where f anf g are arbitrary functions, show that $u_{xx} + u_{yy} = 0$.

8. Solve z^4q^2 - $z^2p = 1$.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. If a_1, a_2, ..., an are the positive zeros of the Bessel function $J_P(t)$ then deduce that $\int_0^1 t\ J_p(a_m t)\ J_p(a_n t)dt = \begin{cases} 0, & if m \neq n \\ \frac{1}{2}\ J_{p+1}^2(a_n), & if m = n \end{cases}$.

Contd...

10. Analyse whether the matrix $\emptyset(t) = \begin{bmatrix} e^{-3t} & te^{-3t} & (\frac{t^2}{2!})e^{-3t} \\ 0 & e^{-3t} & te^{-3t} \\ 0 & 0 & e^{-3t} \end{bmatrix}$ is a fundamental matrix for the system $\overline{x}' = A(t)\overline{x}$ where $\overline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$.

11. Use Charpit's method to solve the partial differential equation
    i) $z^2 = pqxy$ ii) $(p^2 + q^2)y = qz$

12. Find the canonical form of the second order PDE $4\frac{\partial^2 u}{\partial x^2} + 9\frac{\partial^2 u}{\partial y^2} - 6\frac{\partial^2 u}{\partial x \partial y} = 0$

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. Show that in Picard's theorem the error $x(t)$ - $x_n(t)$ satisfies the estimate

$|x(t) - x_n(t)| \leq \frac{L(Kh)^{n+1}}{K(n+1)!}\, e^{Kh}$, $t \in [t_0, t_0 + h]$.

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