

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)  
Chromepet, Chennai - 600 044.

M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025  
SEMESTER - III

**20PAMCT3007 - Complex Analysis**

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

**Section B**

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. If  $f$  is a bounded entire function, then prove that  $f$  is constant.
2. State and prove open mapping theorem.
3. State and prove Residue theorem.
4. State and prove third version of Maximum Modulus theorem.
5. Let  $Re z_n > 0$  for all  $n \geq 1$ . Then  $\prod_{n=1}^{\infty} z_n$  converges to a non-zero number iff the series  $\sum_{n=1}^{\infty} \log z_n$  converges.
6. If  $u : G \rightarrow \mathbb{C}$  is harmonic then prove that  $u$  is infinitely differentiable.
7. Show that, If  $G$  is a bounded Dirichlet region then for each  $a$  in  $G$  there is a Green's function on  $G$  with singularity at  $a$ .
8. Let  $f$  be an entire function of finite order  $\lambda$  where  $\lambda$  is not an integer, then prove that  $f$  has infinitely many zeros.

**Section C**

I - Answer any **TWO** questions ( $2 \times 10 = 20$  Marks)

9. State and prove first version of Cauchy's integral formula.
10. If  $f$  has an isolated singularity at  $a$  then the point  $z = a$  is a removable singularity iff  $\lim_{z \rightarrow a} (z - a)f(z) = 0$ .
11. Let  $G$  be a region
  - (a) The metric space  $Har(G)$  is complete
  - (b) If  $\{u_n\}$  is a sequence in  $Har(G)$  such that  $u_1 \leq u_2 \leq \dots$  then either  $u_n \rightarrow \infty$  uniformly on compact subsets of  $G$  or  $\{u_n\}$  converges in  $Har(G)$  to a harmonic function.
12. Prove that, If  $f$  is an entire function that omits two values then  $f$  is a constant.

II - Compulsory question ( $1 \times 10 = 10$  Marks)

13. State and prove the Weierstrass Factorization theorem.

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