

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.

M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - II

20PAMCT2004 - Algebra - II

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. If a, b in K are algebraic over F then show that $a \pm b$, ab and a/b (if $b \neq 0$) are all algebraic over F .
2. Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F .
3. Show that a polynomial of degree n over a field can have atmost n roots in any extension field.
4. If $p(x)$ is irreducible in $F[x]$ and if v is a root of $p(x)$, prove that $F(v)$ is isomorphic to $F'(w)$ where w is a root of $p(t)$. Also show that this isomorphism σ can so be chosen that
 - i. $v\sigma = w$
 - ii. $\alpha\sigma = \alpha'$ forevery $\alpha \in F$.
5. Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.
6. If K is a field and if $\sigma_1, \dots, \sigma_n$ are distinct automorphisms of K , then prove that it is impossible to find elements a_1, \dots, a_n not all 0, in K such that $a_1 \sigma_1(u) + a_2 \sigma_2(u) + \dots + a_n \sigma_n(u) = 0$ for all $u \in K$.
7. If K is a finite extension of F , then $G(K, F)$ is a finite group and its order, $o(G(K, F))$ satisfies $o(G(K, F)) \leq [K : F]$.
8. Prove that there exists a subspace W of V , invariant under T , such that $V = V_1 \oplus W$.

Section C

I - Answer any **TWO** questions ($2 \times 10 = 20$ Marks)

9. If $p(x)$ is a polynomial in $F[x]$ of degree $n \geq 1$ and is irreducible over F , then show that there is an extension E of F , such that $[E : F] = n$, in which $p(x)$ has a root.

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10. If F is of characteristic 0 and if a, b are algebraic over F , then show that there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.
11. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F .
12. If $T \in A(V)$ has all its characteristic roots in F , prove that there is a basis of V in which the matrix of T is triangular.

II - Compulsory question (1 × 10 = 10 Marks)

13. Prove that the number e is transcendental.
