SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - II
20PAMCT2004 - Algebra - II

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If a, b in K are algebraic over F then justify that $a \pm b$, ab, $a|b$ (if$b \neq 0$) are algebraic over F. In other words, the elements in K which are algebraic over F forma subfield of K.
2. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Also conclude that $[L : F] = [L : K][K : F]$.
3. Validate the statement: A polynomial of degree n over a field can have at most n roots in any extension field.
4. Define derivative of $f(x)$. Let $f(x) \in F[x]$ be of degree $n \geq 1$. Then conclude that there is an extension E of F of degree at most $n!$ in which $f(x)$ has n roots.
5. Define fixed field of G. Prove that the fixed field of G is a subfield of K.
6. Let K is a normal extension of F iff K is the splitting field of some polynomial over F.
7. Define Linear Span. If $T \in A(V)$ has all its characteristic roots in F, then there is a basis of V in which the matrix of T is triangular.
8. Define Index of Nilpotence. If $T \in A(V)$ is nilpotent, then $\alpha_0 + \alpha_1 T + ... + \alpha_m T^m$, where the $\alpha_1 \in F$, is invertible if $\alpha_0 \neq 0$.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Devise a detailed proof that the number e must be transcendental.
10. Define Splitting field for $f(x)$ over F. If $p(x)$ is irreducible in $F[x]$ and if v is a root of $p(x)$ then $F(v)$ is isomorphic to $F'(w)$ where w is a root of $p'(x)$; moreover this isomosphism σ can be chosen that

    (i) $v\sigma = w$, (ii) $\alpha\sigma = \alpha'$ for every $\alpha \in F$.

Contd...

11. Let K be a normal extension of F and let H be a subgroup of the group $G(K, F)$ of automorphisms of K relative to H.
    Let $K_H = \{x \in K/\sigma(x) = x$ for all $\sigma \in H\}$ be the fixed field of H. Then, formulate the proof of the following
    (i) $[K : K_H] = O(H)$ (ii) $H = G(K, K_H)$.
12. Devise a detailed proof for the fact that there exists a subspace W of V invariant under T such that $V = V_1 \oplus W$.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. Define invariant under T. Two nilpotent linear transformations are similar if and only if they have the same invariants

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