

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)  
Chromepet, Chennai - 600 044.

M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025  
SEMESTER - IV

**24PAMET4A04 - Calculus of Variations and Integral Equations**

Total Duration : 2 Hrs. 30 Mins.

Total Marks : 60

**Section B**

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. Derive the Euler's equation for a functional of the form  $\int F(x, y, y')dx$ .
2. Describe variation and its properties .
3. Test for an extremum the functional  $\int_0^{x_1} \frac{\sqrt{1+y'^2}}{y} dx$  given that  $y(0) = 0$  and  $y_1 = x_1 - 5$ .
4. Explain the significance of extremals with corners in variational problems.
5. Solve the integral equation  $g(s) = f(s) + \lambda \int_0^1 (1 - 3st)g(t)dt$ .
6. State and prove Fredholm alternative theorem.
7. Explain Volterra integral equation and singular integral equation.
8. Solve the integral equation  $\int_0^s \frac{g(t)dt}{(s-t)^{1/2}}$ .

**Section C**

I - Answer any **TWO** questions ( $2 \times 10 = 20$  Marks)

9. Extend Euler's equation to functionals involving multiple dependent variables and their first derivatives.
10. Explain the sufficient conditions for an extremum in variational problems and define the function  $E(x, y, p, y')$
11. Solve the Volterra integral equation  $g(s) = 1 + \int_0^s stg(t)dt$ .

**Contd...**

12. Discuss the solution of Abel Integral equation

II - Compulsory question (1 × 10 = 10 Marks)

13. Show that the integral equation  $g(s) = f(s) + \frac{1}{\pi} \int_0^{2\pi} \sin(s+t)g(t)dt$  possess no solution for  $f(s) = s$  and it possess many solution when  $f(s) = 1$ .

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