SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - I
22PAMET1001 - Probability and Distributions

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. The number of female insects in a given region follows a Poisson distribution with mean λ. The number of eggs laid by each insect is a $P(\mu)$ random variable. Find the probability distribution of the number of eggs in the region.
2. Obtain mean and variance of poisson distribution.
3. Let X_1 and X_2 be independent random variables with common density given by

   $$f(x) = \begin{cases} 1, & if 0 < x < 1 \\ 0, & Otherwise \end{cases}.$$

   Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1$ - X_2. Obtain the joint density of Y_1 and Y_2.
4. Let f and g be probability density function with corresponding density functions F and G. Let $h(x, y) = f(x)g(y)[1+\alpha(2F(x)-1)\ (2G(y)-1)]$, Where $|\alpha| \leq 1$ is a constant. Compute the Moment generating function of $Z = X + Y$.
5. Compute the moment generating function of chi-square distribution.
6. Evaluate mean and variance for F distribution.
7. Let be a sequence of Density function defined by $F_n(x) = \begin{cases} 0, & x < 0 \\ 1 - \frac{1}{n}, & 0 \leq x < n \\ 1, & n \leq x \end{cases}.$

   Show that convergence in distribution does not imply convergence of moments.
8. Let $X_n \xrightarrow{P} X$, and g be a continuous function defined on R.
   Then show that $g(X_n) \xrightarrow{P} g(X)$ as $n \to \infty$.

Contd...

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Let X_1, X_2, ... be a sequence of IId random variables having common exponential density with parameter $\beta > 0$. Let $S_n = \sum_{K=1}^{n} X_K$ be the nth partial sum, $n = 1, 2, ...$ and suppose that $t > 0$. If $Y =$ number of $S_n \in [0, t]$, then show that Y is a random variable with $P(t/\beta)$.

10. An urn contains three red and two green balls. A random sample of two balls is drawn (a) with replacement, and (b) without replacement. Let
    $X = 0$ if the first ball is green,
    $X = 1$ if the first ball is red,
    $Y = 0$ if the second ball is green,
    $Y = 1$ if the second ball is red.
    Find the joint probability mass function of (X, Y), the conditional probability mass functions and the conditional expectations.

11. Define bivariate normal distribution. Obtain marginal probability density function of X, Y and Correlation coefficient between X and Y.

12. Let X_1, X_2, ... X_n be iid $N(\mu, \sigma^2)$ random variables.
    Then show that $\overline{X}$ and $(X_1 - \overline{X}, X_2 - \overline{X}, ... X_n - \overline{X})$ are independent.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove Lindberg-Levy Central Limit Theorem.

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