SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
M.Sc.Applicable Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - I
20PAMCT1002 - Real Analysis

Total Duration : 2 Hrs. 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Let $\{f_n\}$ be a sequence of measurable functions defined on the same measurable set. Then show that

   (i) $\sup\limits_{1 \leq i \leq n} f_i$ is measurable for each n

   (ii) $\inf\limits_{1 \leq i \leq n} f_i$ is measurable for each n

   (iii) $\sup f_n$ is measurable.

   (iv) $\inf f_n$ is measurable.

2. Illustrate the sequence of functions $\{f_n\}$, defined on E, converges uniformly on E if and only if for every $\in > 0$ there exists an integer N such that $m \geq N$, $n \geq N$, $x \in E$ implies $|f_n(x) - f_m(x)| \leq \in$.

3. Suppose E and f are as in differentiable in E, $x \in E$, and $\lim_{h \to 0} \dfrac{|f(x+h) - f(x) - Ah|}{|h|} = 0$ holds with $A = A_1$ and with $A = A_2$. Then prove $A_1 = A_2$.

4. Suppose $\sum c_n$ converges. Put $f(x) = \sum\limits_{n=0}^{\infty} c_n \ x^n (-1 < x < 1)$.

   Then $\lim\limits_{x \to 1} f(x) = \sum\limits_{n=0}^{\infty} c_n$.

5. Let f be a bounded measurable function defined on the finite interval (a, b). Show that $\lim\limits_{\beta \to \infty} \displaystyle\int_a^b f(x) \sin \beta x dx = 0$.

6. If K is a compact metric space, if $f_n \in b(K)$ for $n = 1, 2, 3, ...$ and if $\{f_n\}$ converges uniformly on K, then classify $\{f_n\}$ is equi continuous on K.

7. Suppose f maps a convex open set $E \subset R^n$ into R^m, f is differentiable in E, and there is a real number M such that $\|f'(x)\| \leq M$ for every $x \in E$. Then prove that $|f(b) - f(a)| \leq M|b - a|$ for all $a \in E$, $b \in E$.

Contd...

8. If, for some x, there are constants $\delta > 0$ and $M < \infty$ such that $|f(x+t) - f(x)| \leq M|t|$ for all $t \in (-\delta, \delta)$, then examine $\lim\limits_{N \to \infty} S_N(f; x) = f(x)$.

Section C

I - Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Show that the class M is a σ-algebra.
10. State and Prove Fatou's lemma.
11. State and Prove Stone-Weierstrass theorem.
12. If X is a complete metric space, and if ϕ is a contraction of X into X, then examine there exists one and only one $x \in X$ such that $\phi(x) = x$.

II - Compulsory question $(1 \times 10 = 10$ Marks)

13. State and Prove Parseval's theorem.

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