

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc.Statistics - END SEMESTER EXAMINATIONS - APRIL 2025

SEMESTER - II

**20USTAT2002 - Allied Mathematics - II**

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

### Section B

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. Show that the countable union of countable sets is countable.
2. Let  $f$  and  $g$  be continuous functions on the closed bounded interval  $[a, b]$  with  $g(a) \neq g(b)$ . If both  $f$  and  $g$  have a derivative at each point of  $(a, b)$  and  $f'(t)$  and  $g'(t)$  are not both equal to zero for any  $t \in (a, b)$  then there exists a point  $c \in (a, b)$  such that  $\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$ .
3. Evaluate  $L(e^t \cos^3 t)$ .
4. Evaluate  $L^{-1} \left[ \frac{4s + 5}{(s - 1)^2(s + 2)} \right]$ .
5. Show that a non decreasing sequence which is bounded above is convergent.
6. Prove that every convergent sequence is bounded. What about it converse ?
7. If  $L(t \sin \omega t) = \frac{2\omega s}{(s^2 + \omega^2)^2}$ , then evaluate  $L(t\omega \cos \omega t + \sin \omega t)$ .
8. Show that  $L^{-1} \left[ \log \left( 1 + \frac{\omega^2}{s^2} \right) \right] = 2 \frac{(1 - \cos \omega t)}{t}$ .

### Section C

Answer any **THREE** questions ( $3 \times 10 = 30$  Marks)

9. Prove that the set  $[0, 1]$  is uncountable.
10. If  $\{a_n\}_{n=1}^{\infty}$  is a sequence of positive numbers such that
  - i)  $a_1 \geq a_2 \geq \dots a_n \geq a_{n+1} \dots$  (that is  $\{a_n\}_{n=1}^{\infty}$  is non increasing
  - ii)  $\lim_{n \rightarrow \infty} a_n = 0$ , then show that the alternating series  $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$  is convergent.
11. State and prove Rolle's theorem.

Contd...

12. Show that

$$\text{i) } L(\cos at - \frac{1}{2}at \sin at) = \frac{s^3}{(s+a^2)^2}.$$

$$\text{ii) } L\left[\frac{1}{2}(\sin at - at \cos at)\right] = \frac{a^3}{(s^2 + a^2)^2}.$$

13. Find the inverse Laplace transformations of

$$\text{i) } \frac{1}{s^2(s^2 + 81)}.$$

$$\text{ii) } \frac{1}{s^2(s^2 + 4)}.$$

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