

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc.Statistics - END SEMESTER EXAMINATIONS - APRIL 2025

SEMESTER - II

20USTCT2004 - Matrix Algebra

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Show that if A and B are idempotent then $A + B$ will be idempotent if and only if $AB = BA = 0$.
2. Show that inverse of a symmetric matrix is a symmetric.
3. State the conditions under which a system of m non-homogeneous linear equations in n unknowns will have
 - (i) no solution
 - (ii) unique solution
 - (iii) infinite number of solutions
4. Prove that if X_i and X_j are the characteristic vectors corresponding to two distinct characteristic roots λ_i and λ_j respectively of a $n \times n$ square matrix A then
 - (a) X_i and X_j are always linearly independent.
 - (b) X_i and X_j are orthogonal if A is symmetric.
5. When is a quadratic form set to be
 - (i) Positive definite
 - (ii) Negative definite
 - (iii) Positive semi definite
 - (iv) Negative semi definite
 - (v) Indefinite.
6. Show that if A is an orthogonal matrix then A' and A^{-1} are also orthogonal matrices.
7. Show that $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$
8. State and prove the Cayley-Hamilton theorem.

Contd...

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. Show that every square matrix can be uniquely expressed as $P + iQ$ where P and Q are Hermitian matrices.
10. Apply the properties and prove that elementary transformation of a matrix does not alter its rank.
11. Prove that the necessary and sufficient condition for the system of equations $AX = B$ to be consistent.
12. Apply the suitable properties and prove that any two characteristic vectors corresponding to two distinct characteristic roots of a
 - (i) Hermitian matrix,
 - (ii) Real symmetric matrix
 - (iii) Unitary matrix are orthogonal.
13. Reduce $x^2 + y^2 + 4z^2 + 9t^2 - 2xy - 4yz + 6yt - 6tx - 12tz$ to sum of squares.
