

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc.Statistics - END SEMESTER EXAMINATIONS - APRIL 2025

SEMESTER - I

20USTAT1001 - Allied Mathematics - I

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

- Show that $\frac{1}{3!} + \frac{2}{5!} + \frac{3}{7!} + \dots = \frac{1}{2e}$.
- If $x = r \cos \theta$, $y = r \sin \theta$, $z = z$ (where r, θ, z are cylindrical coordinates) show that $\frac{\partial(x, y, z)}{\partial(r, \theta, z)} = r$.
- Derive the Maclaurin's series for $\cos \theta$.
- Using the reduction formula evaluate $\int_0^{\pi/2} \sin^8 x dx$.
- If $y = \frac{3}{(x+1)(2x-1)}$ find y_n .
- If $u = x^2 + y^2 + z^2$, $v = x + y + z$ and $w = xy + yz + zx$ then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 0$.
- Show that $\sin^6 \theta = \frac{-1}{32} [\cos 6\theta - 6 \cos 4\theta + 15 \cos 2\theta - 10]$.
- Using the reduction formula evaluate $\int_0^{\pi/2} \sin^6 x \cos^9 x dx$.

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

- Sum to infinity the series $\frac{7}{9} + \frac{7.9}{9.12} + \frac{7.9.11}{9.12.15} + \dots$
- If $y = \sin(m \sin^{-1} x)$, then prove that
 - $(1 - x^2)y_2 - xy_1 + m^2y = 0$.
 - $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y = 0$.
- Find the maximum or minimum values of the function $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$.

Contd...

12. Show that

i) $-2^{10} \cos^5 \theta \sin^6 \theta = \cos 11\theta - \cos 9\theta - 5 \cos 7\theta + 5 \cos 5\theta + 10 \cos 3\theta - 10 \cos \theta.$

ii) $-2^{11} \cos^5 \theta \sin^7 \theta = \sin 12\theta - 2 \sin 10\theta - 4 \sin 8\theta + 10 \sin 6\theta + 5 \sin 4\theta - 20 \sin 2\theta.$

13. Derive the formula for $I_{m,n} = \int \sin^m x \cos^n x dx.$
