SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - II
**20UMACT2003 - Classical Algebra**

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

## Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find the sum to infinity of the series $1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + ...$
2. Sum the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + ... \; to \; \infty$
3. Solve the equation $x^4 + 4x^3 + 5x^2 + 2x$ - 2 $=$ 0 given that one of the roots is -1 + $\sqrt{-1}$.
4. Frame an equation with rational coefficients, one of whose roots is $\sqrt{5} + \sqrt{2}$.
5. Remove the fractional coefficients from the equation $x^3 + \frac{1}{4}x^2 - \frac{1}{16}x + \frac{1}{72} = 0$
6. Write a short note on (i) symmetric matrix (ii) skew symmetric matrix (iii) Hermitian and skew Hermitian matrices.
7. Show that A $= \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ is orthogonal.
8. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

## Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Show that $\frac{5}{1.2.3} + \frac{7}{3.4.5} + \frac{9}{5.6.7} + ...\infty = 3log2 - 1.$

**Contd...**

10. Show that $\frac{a^2}{x-\alpha}+\frac{b^2}{x-\beta}+\frac{c^2}{x-\gamma}-x+\delta=0$ has only real roots if a, b, c, $\alpha$, $\beta$, $\gamma$, $\delta$ are all real.

11. Solve the equation $27x^3$ + $42x^2$ - $28x$ - 8 = 0 whose roots are in geometric progression.

12. Prove that any real square matrix A = $\begin{bmatrix} 2 & 1 & 4 \\ 8 & -1 & 3 \\ 3 & -5 & 0 \end{bmatrix}$ may be written as the sum of a symmetric matrix R and a skew symmetric matrix S, where R = $\frac{1}{2}(A+A^t)$ and S = $\frac{1}{2}(A-A^t)$.

13. Find the characteristic equation of the matrix A = $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ and hence determine its inverse using Cayley-Hamilton theorem.

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