

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025

SEMESTER - II

20UMACT2003 - Classical Algebra

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

- Sum the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \dots$ to ∞
- Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ given that one of the roots is $1 - \sqrt{5}$.
- Solve the equation: $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.
- Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.
- i) Find the smallest number with 18 divisors.
ii) Find the number of integers less than n and prime to it when $n = 729$ and 720 .
- Explain Orthogonal matrix and prove any three of its properties.
- Show that $\frac{5}{1.2.3} + \frac{7}{3.4.5} + \frac{9}{5.6.7} + \dots \infty = 3\log 2 - 1$.
- Remove the fractional coefficients from the equation $x^3 + \frac{1}{4}x^2 - \frac{1}{16}x + \frac{1}{72} = 0$.

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

- Sum the series to infinity $\frac{15}{16} + \frac{15.21}{16.24} + \frac{15.21.27}{16.25.32} + \dots$
- Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in Arithmetical progression if $2p^3 - 9pq + 27r = 0$. Show that the above condition is satisfied by the equation $x^3 - 6x^2 + 13x - 10 = 0$. Hence or otherwise solve the equation.
- Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ can be transformed into a reciprocal equation by diminishing the roots by unity. Hence solve the equation.

Contd...

12. Show that any real square matrix A may be written as the sum of a symmetric matrix R and a skew symmetric matrix S ,

where $R = \frac{1}{2}(A+A')$ and $S = \frac{1}{2}(A-A')$ Hence represent the matrix $\begin{bmatrix} 2 & 1 & 4 \\ 8 & -1 & 3 \\ 3 & -5 & 0 \end{bmatrix}$

as the sum of a symmetric matrix and a skew symmetric matrix.

13. i) Show that if x and y are both prime to the prime number n , then show that $x^{n-1} - y^{n-1}$ is divisible by n . Deduce that $x^{12} - y^{12}$ is divisible by 1365.

ii) Show that $(18)! + 1$ is divisible by 437.
