

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025

SEMESTER - VI

20UMACT6014 - Complex Analysis

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. State Cauchy Riemann Equation in Cartesian form and Verify the same for $f(z) = z^2$.
2. Stating Cauchy Riemann Equation in Polar form verify whether $f(z) = 1/z$ satisfies or not.
3. Verify that $f(z) = -ie^{iz}$ is an entire function.
4. State Cauchy–Goursat theorem and Using it evaluate $\int_C \frac{z}{z-3} dz$ when the contour C is the unit circle $|z| = 1$.
5. Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate $\int_C \frac{z}{2z+1} dz$.
6. Derive Cauchy Riemann equation in polar co-ordinate.
7. Let C be the circle $|z| = 3$, described in the positive sense.

Show that if $\int_C \frac{2s^4 - s - 2}{s - z} dz$ then for $|z| \neq 3$, $g(2) = 8\pi i$.

8. Write Taylor series expansion of $f(z)$ about the point z_0 and Expand $f(z) = e^z$ about the point $z = 0$.

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. State the sufficient condition for Differentiability. Discuss the derivatives of
(i) $f(z) = ez$ (ii) $f(z) = |z|^2$.

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10. (i) Prove that $u = y^3 - 3x^2y$ is Harmonic and obtain its Harmonic conjugate.
(ii) Show that $\left| \int_c \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}$; where c is the arc of circle $|z|$ from $z = 2$ to $z = 2i$.
11. State and Prove Cauchy's Integral formula.
12. State and Prove Liouville's theorem. Hence, deduce Fundamental theorem of Algebra.
13. Expand $f(z) = \frac{-1}{(z-1)(z-2)}$ as a Laurent Series in the domains
(i) $|z| < 1$; (ii) $1 < |z| < 2$; (iii) $|z| > 2$.
