SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - VI
20UMACT6014 - Complex Analysis

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Define accumulation point and prove that if a set contains each of its accumulation points, then it must be a closed set.
2. Derive Cauchy-Riemann equation in polar coordinates.
3. Evaluate the integral: $\int_0^\infty e^{-zt}dt$ [Re z $>$ 0].
4. Evaluate: $I = \int\limits_c \bar{z}dz$ where c is the right hand half $z = 2e^{i\theta}$ $\left(\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$.
5. If a function f is entire and bounded in the complex plane, then prove that $f(z)$ is constant throughout the plane.
6. Suppose that $Z_n = x_n + iy_n$ and $Z = x + iy$. Then prove that $\lim\limits_{n\to\infty} Z_n = Z$ if and only if $\lim\limits_{n\to\infty} x_n = x$ and $\lim\limits_{n\to\infty} y_n = y$.
7. Illustrate the statement: The power series $S(Z) = \sum\limits_{n=0}^{\infty} a_n(z - z_0)^n$ can be differentiated term by term, that is, at each point z interior to the circle of convergence of that series is $S'(z) = \sum\limits_{n=1}^{\infty} na_n(z - z_0)^{n-1}$.
8. Evaluate the improper integral $\int_0^\infty \frac{x^a}{(x^2+1)^2}dx$, where $-1 < a < 3$ and $x^a = exp(a\ lnx)$.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Suppose $f(z)$=$u(x, y)$+$iv(x, y)$ and $z = x_0+iy_0$, $w_o = u_0+iv_0$, then prove that $\lim\limits_{z\to z_0} f(z)$=$w_0$ if and only if $\lim\limits_{(x,y)\to(x_0,y_0)} u(x, y) = u_0$ and $\lim\limits_{(x,y)\to(x_0,y_0)} v(x, y) = v_0$

Contd...

10. State and prove Cauchy- Goursat theorem.

11. Illustrate briefly that suppose $f(z)$ is entire and the harmonic function $u(x, y) = Re(f(z))$ has an upper bound u_0, that is $u(x, y) \leq u_0$ for all points (x, y) in the xy plane then prove that $u(x, y)$ must be constant throughout the plane.

12. If a series $\sum_{n=0}^{\infty} a_n(z - z_0)^n$ converges to $f(z)$ at all points interior to some circle $|z - z_0| = R$ then prove that it is the Taylor series expansion for f in powers of $z - z_0$.

13. Use residues to evaluate $\int\limits_0^{2\pi} \frac{d\theta}{1 + a\sin\theta}$, $-1 < a < 1$

*****