SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - VI
20UMACT6013 - Linear Algebra

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If $\{v_1, v_2, v_3 ... v_n\}$ is a basis of V a vector space over F and if $\{w_1, w_2, w_3 ... w_m\} \in V$ are linearly independent over F , prove that $m \leq n$.
2. If V is finite-dimensional and $v \neq 0 \in V$, then prove that there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.
3. Derive Schwarz inequality.
4. Prove that the element $\lambda \in F$ is a characteristics root of $T \in A(V)$ if and only if for some $V \neq 0$ in V, vT = λv.
5. If V is n–dimensional over F and if $T \in A(v)$ has the matrix $m_1(T)$ in the basis $v_1, v_2, ..., v_n$ and the matrix $m_2(T)$ in the basis $w_1, w_2, ..., w_n$ of V over F, then prove that there is an element $C \in F_n$ such that $m_2(T) = Cm_1(T)C^{-1}$.
6. Prove that the vectors (1, 2, 1), (2, 1, 0) and (1, –1, 2) are linearly independent.
7. If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if T maps V onto V.
8. If a, b, c are real numbers such that $a > 0$ and $a\lambda^2 + 2b\lambda + c \geq 0$ for all real numbers λ, prove that $b^2 \leq ac$.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If V is finite dimensional and if W is a subspace of V, then prove that
   (a) $dimW \leq dimV$ and (b) $dim(V/W) = dimV - dimW$.
10. If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that λ is a root of the minimal polynomial of T. In particular, T only has a finite number of characteristic root in F.
11. Let V be a finite dimensional inner product space then prove that V has an orthonormal set as a basis.

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12. Prove that if V is n -dimensional over F and if $T \in A(V)$ has all its characteristic roots in F ,then T satisfies a polynomial of degree n over F.

13. Define linear span of s. also prove that:

    a) $L(s)$ is a subspace of v.

    b) If $v_1, v_2, ...v_n \in v$ are linearly independent then prove that every element in their $L(s)$ has a unique representation in the form $\lambda_1 v_1 + ... + \lambda_n v_n$ with $\lambda_i \in F$.

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