

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.

B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - V

20UMACT5012 - Graph Theory

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. I. Prove that the sum of the degrees of the points of a graph G is twice the number of lines
II. In any graph G prove that the number of points of odd degree is even.
2. Prove that every Hamiltonian graph is 2-connected graph.
3. If G is a tree then prove that every two points of G are joined by a unique path.
4. Prove that K_5 is non-planar.
5. Define
 - a. Vertex colouring
 - b. Edge colouring
 - c. Chromatic number with an example.
6. If G is connected plane graph having V, E and F as the sets of vertices, edges and faces respectively then $|V| - |E| + |F| = 2$.
7. Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.
8. Prove that in a graph any $u - v$ walk contains a uv path.

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. Prove that the maximum number of lines among all p point graphs with no triangles is $\left\lfloor \frac{p^2}{4} \right\rfloor$.
10. State and prove Dirac's theorem.
11. A graph G is a bipartite graph if and only if it contains no odd cycles.
12. A graph can be embedded in the surface of the sphere if and only if it can be embedded in a plane.
13. If G is simple graph with largest vertex-degree, then G is colourable.
