

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025

SEMESTER - V

20UMACT5010 - Real Analysis

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Show that the set $[0, 1] = \{x : 0 \leq x \leq 1\}$ is uncountable.
2. Show that every Cauchy sequence of real numbers is convergent.
3. Determine whether every open ball in a metric space (M, d) is an open set.
4. State and prove the Generalization of the nested interval theorem in complete metric space.
5. State and prove the Rolle's theorem.
6. Show that a non decreasing sequence which is bounded above is convergent.
7. Compute the limit of $\sqrt{n^2 + 1} - n$ and examine its convergence.
8. Justify that the function $f(x) = |x|$ is continuous at $x = 0$ but not differentiable at $x = 0$

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. Show that the countable union of countable set is countable.
10. State and prove Leibnitz theorem for alternating series.
11. Let (M, ρ) be a metric space and let $a \in M$. Let f and g be real valued functions whose domains are subsets of M . If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = N$, then apply the limit properties to prove the following results
 - (a) $\lim_{x \rightarrow a} f(x) + g(x) = L + N$.
 - (b) $\lim_{x \rightarrow a} f(x) - g(x) = L - N$.
 - (c) $\lim_{x \rightarrow a} f(x)g(x) = LN$.
12. Show that a bounded infinite set of real numbers has atleast one limit point.
13. State and prove the first Fundamental Theorem of Calculus.
