SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - V

20UMACT5009 - Modern Algebra

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. a) If n is a positive integer and a is relatively prime to n, then prove that $a^{(n)} \equiv 1(\text{modn})$.
   b) Show that HK is a subgroup of G if and only if $HK = KH$.
2. Prove that a homomorphism ϕ of G into $\overline{G}$, with kernel $K\phi$ is an isomorphism of G into $\overline{G}$ if and only if $K\phi = (e)$.
3. If G is a group then show that $A(G)$ the set of automorphisms of G is also a group.
4. Prove that a finite integral domain is a field.
5. a) Define ideal of the ring R.
   b) If U is an ideal of R and $I \in U$, then prove that $U = R$.
6. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.
7. Illustrate the following statement that the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R.
8. Prove that if R be a Euclidean ring, then every element in R is either a unit in R or can be written as the product of a finite number of prime elements of R.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. a) Define normal subgroup, Quotient group with example.
   b) Prove that if G is a finite group and N is a normal subgroup of G, then $O\left(\frac{G}{N}\right) = \frac{O(G)}{O(N)}$.
10. Illustrate that every group is isomorphic to a subgroup of $A(S)$ for some appropriate S.

Contd...

11. a) Define kernal of a function ϕ.
    b) If ϕ is a homomorphism of R into R' with kernel $I(\phi)$, then prove that
        (i) $I(\phi)$ is a subgroup of R under addition.
        (ii) If $a \in I(\phi)$ and $r \in R$ then both ar and ra are in $I(\phi)$.

12. Prove that, let R be a commutative ring with unit element whose only ideals are (0) and R itself.

13. Prove that every integral domain can be imbedded in a field.

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