

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025

SEMESTER - IV

**20UMACT4007 - Vector Calculus and Fourier Transforms**

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

### Section B

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

- Find the directional derivative of  $xyz - xy^2 z^3$  at the point  $(1, 2, -1)$  in the direction of the vector  $\vec{i} - \vec{j} - 3\vec{k}$ .
- Prove that  $\text{div } \vec{r} = 3$  and  $\text{curl } \vec{r} = 0$ , where  $\vec{r}$  is the position vector.
- If  $\vec{F} = 3xy \vec{i} - y^2 \vec{j}$ , evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $C$  is the curve on the  $xy$  plane  $y = 2x^2$  from  $(0, 0)$  to  $(1, 2)$ .
- Evaluate  $\iiint \text{div } \vec{F} dV$ , where  $\vec{F} = x \vec{i} + y \vec{j} + z \vec{k}$  and bounded by the planes  $x = 0, x = a, y = 0, y = a, z = 0, z = a$ .
- If  $F(f(x)) = F(s)$  then prove that  $F(f(ax)) = \frac{1}{a} F\left(\frac{s}{a}\right)$ .
- Find the Fourier Sine transform of  $e^{-ax}$ .
- Find the Fourier Cosine transform of  $f(x) = \begin{cases} 1, & 0 < x < a \\ 0, & x \geq 0 \end{cases}$
- Find the Fourier Sine transform of  $\frac{1}{x}$ .

### Section C

Answer any **THREE** questions ( $3 \times 10 = 30$  Marks)

- Find the equation of the tangent plane and normal to the surface  $xyz = 4$  at the point  $(1, 2, 2)$ .
- If  $\vec{F} = 4xz \vec{i} - y^2 \vec{j} + yz \vec{k}$ , evaluate  $\iint_S \vec{F} \cdot \vec{n} ds$ , where  $S$  is the surface bounded by the planes  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ .

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11. Verify Stokes theorem for  $\vec{F} = (2x - y) \vec{i} - yz^2 \vec{j} - y^3z \vec{k}$ , where  $S$  is the upper half of the sphere  $x^2 + y^2 + z^2 = 1$  and  $C$  its boundary.

12. Find the Fourier transform of  $f(x) = \begin{cases} x, & |x| < a \\ 0, & |x| > a \end{cases}$

13. Use Parseval's identity to evaluate  $\int_0^\infty \frac{dx}{(x^2 + a^2)^2}$  and  $\int_0^\infty \frac{x^2 dx}{(x^2 + a^2)^2}$ .

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