SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai - 600 044.
B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025
SEMESTER - II
20UMACT2004 - Integral Calculus and Fourier Series

Total Duration : 2 Hrs.30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Apply the reduction formula, find the value of the following
   i) $\int_0^{\frac{\pi}{2}} \sin^7 x dx.$ ii) $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^9 x \, dx.$
2. Apply the Bernouli's formula, evaluate the following
   i) $\int_0^{\frac{\pi}{2}} x^4 \, e^x dx.$ ii) $\int_0^{\frac{\pi}{2}} x^3 \, \cos 2x dx.$
3. Find the Fourier Coefficient a_n for $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ \pi - x, & \pi \leq x \leq 2\pi \end{cases}.$
4. Find the Fourier Coefficient b_n for $f(x) = (\pi - x)^2$, $-\pi < x < \pi$.
5. Prove that $\Gamma(n+1) = n!$, provided that n is a positive integers.
6. Show that $\beta(m, n) = \beta(n, m)$.
7. Show that $\int_0^{\frac{\pi}{2}} \sqrt{\cot \theta d\theta} = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right).$
8. Find the Fourier series for the function $f(x) = (\pi - x)^2$, -$\pi < x < \pi$.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If $I_n = \int x^n \cos ax dx$ then prove that
   $$I_n = \frac{x^n \sin ax}{a} + \frac{n}{a^2} \, x^{n-1} \sin ax - \frac{n(n-1)}{a^2} I_{n-2}.$$
10. Apply the Gamma function, show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$
11. Prove that $\beta(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$

Contd...

12. Obtain the Fourier Series for the function $f(x) = \frac{1}{2}(\pi - x)$, $0 < x < 2\pi$ and hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{4}$.

13. Find the Fourier series for $f(x) = \begin{cases} x, & -\pi \leq x \leq 0 \\ -x, & 0 \leq x \leq \pi \end{cases}$.

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$.

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