

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai - 600 044.

B.Sc.Mathematics - END SEMESTER EXAMINATIONS - APRIL 2025

SEMESTER - II

20UMACT2004 - Integral Calculus and Fourier Series

Total Duration : 2 Hrs.30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Evaluate $\int x^4(\log x)^3 dx$.
2. Integrate $e^x \sin 2x$.
3. By transforming into polar co-ordinates evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ ($b > a$).
4. Evaluate $\int_0^\infty e^{-x^2} dx$.
5. Evaluate $\int_0^1 x^m \left(\log \frac{1}{x} \right)^n dx$.
6. Show that $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^\infty (-1)^n \frac{\cos nx}{n^2}$ in the interval $(-\pi \leq x \leq \pi)$.
7. If $f(x) = \begin{cases} x & \text{when } 0 < x < \frac{\pi}{2} \\ \pi - x & \text{when } x > \frac{\pi}{2} \end{cases}$

Expand $f(x)$ as a sine series in the interval $(0, \pi)$.

8. Express $f(x) = c - x$ where $0 < x < c$ as a half range cosine series with period $2c$.

Section C

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. Integrate $x^2(e^x + e^{-x})$.
10. Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy dx dy$ and evaluate it.

Contd...

11. Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of gamma functions and evaluate the integral

$$\int_0^1 x^5(1-x^3)^{10} dx.$$

12. A function $f(x)$ is defined within the range $(0, 2\pi)$ by the relations

$$f(x) = \begin{cases} x & \text{in } (0, \pi) \\ 2\pi - x & \text{in } (\pi, 2\pi) \end{cases}$$

Express $f(x)$ as a Fourier series in the range $(0, 2\pi)$.

13. Find a Fourier series with period 3 to represent $f(x) = 2x - x^3$ in the range $(0,3)$.
