SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS NOVEMBER-2022
SEMESTER - I
**22UAIAT1001 - Allied Mathematics - I**

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

## Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find the sum of the series to infinity:
   $1 + \frac{1}{3} + \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \ldots$
2. Find the positive root of $f(x) = 2x^3$ - $3x$ - 6 = 0 by Newton-Raphson method correct to five decimal places.
3. Using Cayley-Hamilton theorem, calculate $A^4$ when $A = \begin{bmatrix} -1 & 3 \\ -1 & 4 \end{bmatrix}$.
4. Show that the matrix $A = \begin{bmatrix} -2 & 1 & 4 \\ 8 & -1 & 3 \\ 3 & -5 & 0 \end{bmatrix}$ can be written as the sum of symmetric matrix R and a skew symmetric matrix S,
   where $R = \frac{1}{2}(A + A^t)$ and $S = \frac{1}{2}(A - A^t)$.
5. Diminish the roots of the equation $x^4$ - $4x^3$ - $7x^2 + 22x + 24 = 0$.
6. Express $\frac{\sin 6\theta}{\sin \theta}$ in terms of $\cos\theta$.
7. If cosh $u$ = sec $\theta$, show that $u = \log \tan(\frac{\pi}{4} + \frac{\theta}{2})$.
8. Find $y_n$, where $y = \frac{x^2}{(x-1)^2(x+2)}$

## Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. (i) State the relation between E and $\Delta$.
   (ii) Using Lagrange's interpolation formula find y(10) from the following table:

| **x:** | 5 | 6 | 9 | 11 |
|---|---|---|---|---|
| **y:** | 12 | 13 | 14 | 16 |

**Contd...**

10. Determine the eigenvalues and eigenvectors for the given matrix A $= \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 8 & 1 & 1 \end{bmatrix}$

11. Solve the equation $2x^5$ - $15x^4 + 37x^3$ - $37x^2 + 15x$ - $2 = 0$.

12. Derive the expansion of $\tan\, n\theta$ in powers of $\tan\theta$.

13. (i) If $x =$ r $\sin\theta\, \cos\phi$, $y =$ r $\sin\theta\, \sin\phi$, $z =$ r $\cos\theta$, then find $\dfrac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$.

    (ii) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ is $4a \cos\dfrac{\theta}{2}$

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