SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) - END SEMESTER EXAMINATIONS APRIL-2023
SEMESTER - VI
20UMACT6014 - Complex Analysis

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Show that if a function f (z) is continuous and nonzero at a point Z_0, then $f(z) \neq 0$ throughout some neighborhood of that point.
2. Justify: If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then its component functions u and v are harmonic in D.
3. Find the value of the integral I $= \int_c \bar{z}dz$ where C is the right-hand half, $z = 2e^{i\theta}\left(\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$
4. Show that if a function is analytic at a point, then its derivatives of all orders exist at that point. Those derivatives are, moreover; all analytic there.
5. State and prove fundamental theorem of algebra.
6. Evaluate the integral $\int_c \frac{5z-2}{z(z-1)}dz$ when C is the circle and |z|=2.
7. Find the linear fractional transformation that maps the points $z_1 = 2$, $z_2 =$ i, $z_3 =$ -2 onto the points $w_1 = 1$, $w_2 =$ i, $w_3 =$ -1.
8. Discuss the transformation w = sin z

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. State and prove, the sufficient condition for differentiability at a point of the function $f(z)$.
10. State and prove Cauchy Integral Formula
11. State and prove Taylor's theorem
12. If a function f is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, then justify $\int_c f(z)dz = 2\pi$i $\times \sum_{i=1}^{n}$ Ri, where Ri's are residue.
13. Criticize Linear Fractional Transformations and its Inverse. Also explain that any linear fractional transformation transforms circles and lines into circles and lines.

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