SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. END SEMESTER EXAMINATION APRIL/NOV – 2021
SEMESTER – I
20PAMCT1002 – Real Analysis

Total Duration : 3 hrs	**Total Mark : 75**
MCQ : 30 min	MCQ : 15
Descriptive : 2 Hrs. 30 Mins.	Descriptive : 60

Section A

Answer any ***SIX*** questions (6 x 5 =30)

1. For any sequence of sets $\{E_i\}$, Prove that $m^*(\bigcup_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} m^*(E_i)$.
2. State and prove Riemann Lebesgue lemma.
3. Let α be monotonically increasing on [a, b]. Suppose $f_n \epsilon\, R(\alpha) on$ [a, b], for n=1,2,... and suppose $f_n \to f$ uniformly on [a, b]. Then prove that $f \epsilon\, R(\alpha)$ on [a, b] and $\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.
4. Suppose f maps a convex open set $E \subset R^n$ into R^m, f is differentiable in E, and there is a real number M such that $\| f'(x) \| \leq M$ for every $x \epsilon E$. Then prove that $|f(b)-f(a)| \leq M |b-a|$ for all $a,b \epsilon E$.
5. If x>0 and y>0, then show that $\int_0^1 t^{x-1}(1-t)^{y-1} dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.
6. show that if $m^*(E)=0$ then E is measurable.
7. Show that $\int_1^{\infty} \frac{dx}{x} = \infty$.
8. Let $\{\phi_n\}$ be orthonormal on [a,b]. Let $s_n(x) = \sum_{m=1}^{n} c_m \phi_m(x)$ be the n^{th} partial sum of the Fourier series of f, and suppose $t_n(x) = \sum_{m=1}^{n} \gamma_m \phi_m(x)$. Then prove $\int_a^b |f - s_n|^2 dx \leq \int_a^b |f - t_n|^2\, dx$ and equality holds if and only if $\gamma_m = c_m$ (m=1,...n).

Contd…..

Section B

Part A

Answer any ***TWO*** questions (2 x 10 =20)

9. Prove that the class M is a σ-algebra.
10. State and prove Stone-Weierstrass theorem.
11. State and prove Inverse function theorem.
12. State and prove Implict function theorem.

Part B

Compulsory Question (1 x 10 = 10)

13. If f is a Riemann integrable and bounded over the finite interval [a, b], prove that f is integrable and R $\int_a^b f\,dx = \int_a^b f\,dx$.