B.Sc DEGREE EXAMINATION,APRIL 2019
III Year V Semester
Real Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Find the l.u.b and g.l.b for the following set $\{\ \pi + 1,\ \pi + \frac{1}{2}, \pi + \frac{1}{3}\ , \ldots\}$
2. Define convergent sequence.
3. Define bounded sequence.
4. Find the limit superior and inferior of 1,2,3,1,2,3,1,2,3,1,2, 3. . . .
5. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.
6. Prove that $\lim_{x \to 1} \frac{x^2\ -1}{x-1}$ =2
7. Define a metric space.
8. Define a open set.
9. If f(x)=x ; 0 $\leq x \leq 1\ and\ \sigma$ =$\{$0, $\frac{1}{2}$,1$\}$compute U[f; $\sigma]$
10. State the Rolle's theorem.
11. When will you say f has a derivative at c?
12. State the first fundamental theorem of calculus.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If The sequence of real number $\{$S$_n\ \}_{n=1}^{\infty}$ is convergent . Then $\{$S$_n\ \}_{n=1}^{\infty}$is bounded
14. If $\sum_{n=1}^{\infty} a_n$ is a convergent series. Then $\lim_{n\to\infty} a_n$ =0.
15. Let R be the set of all real numbers. Let d(x,y)=|x-y|,x,y $\in$R. Prove that d is a metric space.
16. If G$_1$ and G$_2$ are open subset of the metric space M. Then G$_1$ $\cap$G$_2$ is also open.
17. Using the Rolle's theorem find the value of c f(x)=sinx 0$\leq x \leq \pi$.
18. Let f be a bounded function on [a,b]. Then prove that every upper sum for j is greater than or equal to every lower sum for f (i.e. if σ and τ are any two subdivision of [a,b], then prove that U[f ; σ] $\geq$ L[f ; τ])
19. State and prove that The law of the mean.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. The Sequence $\{(1+\frac{1}{n})^n\}_{n=1}^{\infty}$ is convergent.
21. State and prove that the ratio test.
22. Let (M,ρ) be a metric space . Let f and g be real valued function defined on M and a$\in M$ .Let $\lim_{x\to a} f(x) =$ L and $\lim_{x\to a} g(x) = M$

    (a) $\lim_{x\to a}[f(x) + g(x)] = L + M$ (b) $\lim_{x\to a} fg(x) =$ LM
23. Prove that the set R' is of the second category.
24. State and prove that chain rule.

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