Using Quadratic Functions in Vertex Form to Describe Projectile Motion

1) Warm Up: Open the Phet Simulation Projectile Motion

Create several different launches and notice their vertex points. Discuss what the Vertex Form equations for these launches will look like. How might the "a" coefficient in each equation change depending on what the launch looks like?

2) Choose a set of initial conditions and perform a launch. Your launch does not need to hit the target. Record the following information. (Don't forget units!) SAMPLE ANSWER

Initial Height	3 meters	Initial Speed	8 meters/sec
Launch Angle	60 degrees	Object	pumpkin

Look at the path of your projectile. Your goal is to write a quadratic equation in Vertex Form that models the projectile's path. *Your equation will relate the object's horizontal distance (called the range) to its vertical height*.

3) Find the vertex point of the projectile:

Horizontal Distance	2.82 meters	Height	5.45 meters

Write your vertex as a coordinate (horizontal distance, height): (2.82 , 5.45)

4) Next record the total distance that your projectile traveled and its height at that distance:

Horizontal Distance	7.04 meters	Height	0 meters

Write this point as a coordinate (horizontal distance, height): (7.04 , 0)

5) From the sim, draw a sketch of the launch. Be sure to label your axes and any key points.

6) Recall the Vertex Form of a quadratic equation: $f(x) = a(x - h)^2 + k$

Use words to describe what the following variables represent in this context:

x?	Horizontal distance in meters	h?	The x coordinate of the vertex
y?	Vertical height in meters	k?	The y coordinate of the vertex and the maximum height of the trajectory

7) Use the vertex and one other point to find the “a” coefficient for your equation. Round to the hundredths place, if needed.

$$f(x) = a(x - h)^2 + k$$

$0 = a(7.04 - 2.82)^2 + 5.45$

$-5.45 = a(4.22)^2$

$-5.45 \div 17.808 = a$

$a = -0.31$

$f(x) = -0.31(x - 2.82)^2 + 5.45$

Now that you have an “a” coefficient, write the Vertex Form equation:

= Stop and talk

8) What do you notice about the “a” coefficient in your equation? Does this “a” value make sense to you? Explain.

It is a small negative number which makes sense based on my launch. My parabola is upside down and not stretched very tall. It is more compressed.

9) Change your Vertex Form equation into Standard Form: $f(x) = ax^2 + bx + c$

$f(x) = -0.31x^2 + 0.175x + 2.99$

10) What are some things that the Standard Form equation tells you about the projectile launch? Explain any connections that you observe.

It has the same “a” value as the the Vertex Form equation. The “b” value is the initial rate of change. It tells how many meters it travels vertically for every meter it travels horizontally. The “c” value is the initial height, which was 3 meters. My answer is 2.99 because of rounding I did during my calculations.

11) Decide on a way to confirm that your equation(either Vertex Form or Standard Form) correctly models the projectile’s path. Then use your method to verify that the equation is accurate.

I can confirm my answer by picking any other point on the parabola’s graph and plugging it into either equation. I can also use Desmos and graph my equations to see if they match the simulated launch.