

AC Circuits

Purpose: To measure the voltage across various elements connected in an AC circuit

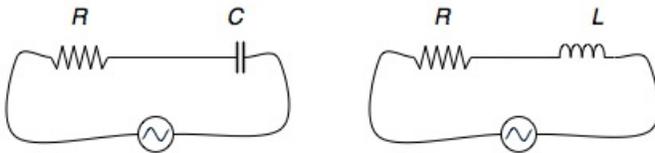
Simulation Used: [Circuit Construction Kit \(AC+DC\)](#) from the PhET at the University of Colorado.

- **Theory.** The capacitive reactance and the inductive reactance of a capacitor and an inductor, respectively, are defined as:

$$X_C = 1/(\omega C) \quad \text{and} \quad X_L = \omega L,$$

where $\omega = 2\pi f$ is the angular frequency in rad/s, and f is the frequency in Hz.

RC circuit and RL circuit.



If the current in the RC circuit is $I(t) = I_o \sin(\omega t)$, then the maximum voltage across each element and the entire circuit is:

$$V_{Ro} = I_o X_R, \quad V_{Co} = I_o X_C, \quad \text{and} \quad V_o = I_o Z,$$

where X_R is simply the resistance R of the resistor, and $Z = \sqrt{R^2 + X_C^2}$ is the impedance of the circuit. The relation between the maximum voltage across each element and the entire circuit is:

$$V_o = \sqrt{V_{oR}^2 + V_{oC}^2}. \quad (\text{Eq.1})$$

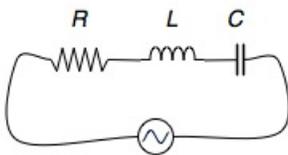
For an RL circuit, similarly, the voltage across the elements and the entire circuit is:

$$V_{Ro} = I_o X_R, \quad V_{Lo} = I_o X_L, \quad \text{and} \quad V_o = I_o Z,$$

with the impedance of the circuit being $Z = \sqrt{R^2 + X_L^2}$. The relation between the voltages then is:

$$V_o = \sqrt{V_{oR}^2 + V_{oL}^2}. \quad (\text{Eq. 2})$$

RLC circuit.



In the case of a resistor, an inductor, and a capacitor connected in series to an AC source (RLC circuit), as shown on the diagram above, the impedance of the circuit is equal to:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}. \quad (\text{Eq. 3})$$

If the current in the circuit is given by $I(t) = I_o \sin(\omega t)$, the voltage across the circuit as a function of time is then given by $V(t) = V_o \sin(\omega t + \phi)$, where ϕ is called a phase angle and for the RLC circuit above is equal to:

$$\tan(\phi) = (X_L - X_C)/(X_R) \quad (\text{Eq.4})$$

Experimentally, the phase angle between the current and the voltage in the circuit, can be determined by measuring the voltage value at the time when the current is zero. If the current $I(t) = I_o \sin(\omega t) = 0$ at some time t , then $\sin(\omega t) = 0$ at that time. Using a trigonometric relation for $\sin(A + B)$, we can write down for the voltage:

$$V(t) = V_o(\sin(\omega t) \cos(\phi) + \cos(\omega t) \sin(\phi)) = V_o \sin(\phi)$$

The phase angle ϕ , then, can be determined as:

$$\phi = \arccos (V(t) / V_o) \quad (\text{Eq.5})$$

• **Activity 1: Voltage across the elements in RC circuit.**

- Connect a resistor and a capacitor in series and connect them to an AC source
 - Click on a resistor, a capacitor and an AC source and drag them into the circuit board.
 - Drag wires and connect the elements in series to form an RC circuit. Two elements are connected when their ends overlap. To disconnect two elements, right-click on the joint and select "split junction".
 - To change the values for an element, right-click over that element and select change value.

R (Ω)	C (F)	E ₀ (V)	f (Hz)
3.0	0.05	10	1.0

- Perform the experiment:
 - **Measuring the maximum voltage using the voltmeter.** If you choose the voltmeter, you can pause the simulation and go step by step over the different voltage values. Record the maximum value you see.
 - **Measuring the maximum voltage on the Voltage Chart.** Click on the Voltage Chart. Measure the amplitude of the voltage on the chart (it's half the height from a trough to a crest)
 - **Taking measurements.** Measure the voltage across the resistor, V_R , the capacitor, V_C , and across both elements, V_{RC} .
- In your lab notebook, write down the data in the following format:

Frequency (Hz)	V_R (V)	V_C (V)	V_{RC} (V)	V_{RC}^2 (V)	$V_R^2+V_C^2$ (V)	% difference
1 Hz						

- Results: Is the sum of the square of the maximum voltages across R and C, $V_R^2+V_C^2$, equal to the square of voltage across both elements, V_{RC}^2 in agreement with Eq.1?

• **Activity 2: Voltage across the elements in RL circuit**

- Connect a resistor and an inductor in series and connect them to an AC source
 - Click on a resistor, inductor and AC source and drag them into the circuit board.
 - Drag wires and connect the elements in series to form an RL circuit. Two elements are connected when their ends overlap. To disconnect two elements, right-click on the joint and select "split junction".
 - To change the values for an element, right-click over that element and select change value.

R (Ω)	L (H)	E ₀ (V)	f (Hz)
50	10	10	1.0

- Perform the experiment:
 - **Taking measurements.** Measure the maximum voltage across the resistor, V_R , the inductor, V_L , and across both elements, V_{RL} .
- In your lab notebook, write down the data in the following format:

Frequency (Hz)	V_R (V)	V_L (V)	V_{RL} (V)	V_{RL}^2 (V)	$V_R^2+V_L^2$ (V)	% difference
1 Hz						

- Results: Is the sum of the square of the maximum voltages across R and L, $V_R^2+V_L^2$, equal to the square of voltage across both elements, V_{RL}^2 in accordance with Eq.2?

• **Activity 3: Impedance of an RLC circuit.**

- Set up the RLC circuit.
 - Click on a resistor, an inductor, a capacitor, and an AC source and drag them into the circuit board.
 - Select an ammeter and voltmeter from the Tools Menu on the right. Connect the Ammeter in series to the RLC circuit.
 - Drag wires and connect the elements in series to form an RLC circuit with an ammeter and AC source.
 - Change the value of the elements to the values given in the table below.

f (Hz)	R (Ω)	L (H)	C (F)	E_0 (V)
0.1	10	10	0.1	10

- Perform the experiment:
 - Measure the maximum voltage across the entire RLC circuit, V_o .
 - Measure the maximum current in the circuit, I_o
 - Measure the value for the voltage when the current in the circuit is zero, $V_{I=0}$
- Data and Results.
 - For the experimental values of the impedance and the phase angle, use the relation $Z=V_o/I_o$, and Eq.5, respectively.
 - For the theoretical value of the impedance and the phase angle, use Eqs.3-4.

V_o (V)	I_o (A)	$V_{I=0}$ (V)	Z_{exp} (Ω)	Z_{th} (Ω)	% difference in Z	ϕ_{exp} in ($^\circ$)	ϕ_{th} in ($^\circ$)	% difference in ϕ

• **Bring your data and results to our next lab!**

Acknowledgements.

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- Some activities are based on the "Laboratory Manual, Physics 231 - 232" by [Walter Wimbush](#), Northern Virginia Community College, 2008.